October Math Gems

Problem of the week 22

§1 Problems

Problem 1.1. If $x^2 = 2023 + y$,$y^2 = 2023 + x$ where $x, y \in R$, $x \neq y$
Then, find the value of xy

Problem 1.2. Evaluate the following series

$$\sum_{n=1}^{1023} \log_2(1 + \frac{1}{n})$$

Problem 1.3. If $\tan(x) - \tan^2(x) = 1$, Then, the value of the following expression is equal to

$$\tan^4(x) - 2\tan^3(x) - \tan^2(x) + 2\tan(x) + 1 =$$

Problem 1.4. The measure of a regular polygon's interior angle is 4 times bigger than the measure of its internal angle. How many sides does the polygon have?

Problem 1.5. How many sides does a convex polygon have if all its external angles are obtuse?

Problem 1.6. show that the triangle ABC where $\frac{a+c}{b} = cot\frac{B}{2}$ is right-angled

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Problem 1.7. Show that, if in triangle ABC we have $\cot(A) + \cot(B) = 2\cot(C)$,then $a^2 + b^2 = 2c^2$

Problem 1.8. Known that

$$\frac{x+y}{2} \geq \sqrt{xy}$$

The minimum value of

$$3^{\sin^6(x)} + 3^{\cos^6(x)}$$

can be written in the form of ab^c. Find $4cb + a$

Problem 1.9. show that in any right-angle triangle ABC we have $\tan\frac{A-B}{2}\tan\frac{C}{2} = \frac{a-b}{a+b}$, where $A + B = C$

Problem 1.10. If

$$\sum_{r=0}^{n-1} \log_2\left(\frac{r+2}{r+1}\right) = \prod_{r=10}^{99} \log_r(r+1)$$

What is the value of n

Problem 1.11. If $(x^2+x+1)+(x^2+2x+3)+(x^2+3x+5)+\cdots+(x^2+20x+39)=4500$ Then, the sum of all possible values of x is

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Problem 1.12. The value of the product

$$\prod_{n=1}^{98} \frac{n^2+2n}{(n+1)^2} = \frac{a}{b}$$

where a, b are two co-prime integers Then, $a+b=$

Problem 1.13.

$$4^{\frac{x}{y}+\frac{y}{x}} = 32$$

$$\log_3(x-y)+\log_3(x+y)=1$$

Find the value of x

Problem 1.14. Evaluate the sum of the expression

$$\sum_{i=1}^{n} i(2)^i$$

Problem 1.15. If $a\sin^2 x + b\cos^2 x = c$, $b\sin^2 y + a\cos^2 y = d$ and $a=b$, then $\frac{a^2}{b^2} =$

Problem 1.16. In a right angle triangle ABC, $\sin^2 a + \sin^2 b + \sin^2 c =$

Problem 1.17. If $\frac{\sin x}{a} = \frac{\cos x}{b} = \frac{\tan x}{c} = k$ then $bc + \frac{1}{ck} + \frac{ak}{1+bk} =$

Problem 1.18. if $a\sin\theta + b\cos\theta = b\sin\phi + a\cos\phi = 1$ and $a\tan\theta = b\tan\phi$ then $a+b =$

Problem 1.19. Evaluate the value of the expression

$$\prod_{a=1}^{89} \tan(a^\circ)$$

Problem 1.20. If $\sin(x)+\sin^2 x+\sin^3 x = 1$,then the value of

$$\cos^6 x - 4\cos^4 x + 8\cos^2 x =$$
